[redacted]

As defined earlier, a discrete random variable takes on any value from a finite set or countably infinite set. Its values are usually obtained [redacted]

The Discrete Uniform Distribution

If an experiment results to a random variable that could take on any of k equally likely possible outcomes, then the random variable is said to have the *discrete uniform distribution.* Experiments such as [redacted]

Pop-Up!

Discrete Uniform Distribution

If X is a discrete random variable that could assume any of the values $x_1, x_2, \ldots, x_k$ with equal probabilities, then X has the *discrete uniform distribution*, denoted by $X \sim \text{Uni}(k)$, with the probability mass function

[redacted]

Example 1

In an experiment of rolling a fair die, let the random variable X be the number of dots in the outcome, which has the equally likely mass [redacted]

[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

The Binomial Distribution and the Bernoulli Distribution

An experiment that results in one of only two possible outcomes, which could be labeled as a success or a failure, is called a *Bernoulli trial*.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

Pop-Up!

Binomial Distribution

The discrete random variable X, which signifies the number of successes in a binomial experiment, has the *binomial distribution*, denoted by $X \sim \text{Bin}(n, p)$, and probability mass function

$$p(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, 2, \ldots, n, \\ 0, & \text{otherwise.} \end{cases}$$

The binomial random variable X has mean $\mu_X = np$ and variance $\sigma_X^2 = np(1-p)$.

Note that if $n = 1$ in a binomial distribution, you will have the *Bernoulli Distribution.*

Pop-Up!

Bernoulli Distribution

A Bernoulli random variable X, denoted by $X \sim \text{Ber}(p) \equiv \text{Bin}(n = 1, p)$, has the probability mass function

$$p(x) = \begin{cases} p^x (1 - p)^{1-x}, & x = 0, 1, \\ 0, & \text{otherwise.} \end{cases}$$

[redacted]

[redacted]

[redacted]

[redacted]

$\approx .55901.$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[[redacted]]

[redacted]

[redacted]

[redacted]

[redacted] √[redacted] √[redacted] [redacted] √[redacted] [redacted] √[redacted] [redacted]

[redacted]

[redacted]

[redacted]

$$[redacted]$$

[redacted]

$$[redacted]$$

[redacted]

$$[redacted]$$

[redacted]

The Hypergeometric Distribution

If the random variable X is defined as the number of successes in a random sample of size n taken without replacement from a population of size N whose elements could be classified into one of two classes, which consists of K successes and $N - K$ failures, then the random variable X has the *hypergeometric distribution*, denoted by $X \sim \text{Hyp}(N, n, K)$.

[redacted]

The following are some examples of experiments that give a discrete random variable with the hypergeometric distribution.

a. counting the number of defective items in a random sample of five [redacted]

Pop-Up!

Hypergeometric Distribution

If the discrete random variable X is defined as the number of successes in a hypergeometric experiment, then it has the hypergeometric distribution with probability mass function

[redacted]

Note that the n trials in a hypergeometric experiment are dependent since sampling is done without replacement, while the sequence of n repeated Bernoulli trials in a binomial experiment are independent.

[redacted]

[redacted]

[redacted]

[redacted]

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[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	0

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

The Negative Binomial Distribution and the Geometric Distribution

If the random variable X is defined as the number of independent and identical Bernoulli trials at which the kth success occurs, then X has the *negative binomial distribution,* denoted by $X \sim \text{NegBin}(k, p)$.

Some examples of activities that have the negative binomial distribution are: counting how many items need to be inspected from [redacted]

Pop-Up!

Negative Binomial Distribution

The discrete random variable X, defined as the number of trials needed at which the kth success occurs, has the *negative binomial distribution* with probability mass function given by

[redacted]

It has mean $\mu_X = \dfrac{k}{p}$ and variance $\sigma_X^2 = \dfrac{k(1-p)}{p^2}$.

You have to remember that in the binomial distribution, the number n of Bernoulli trials is fixed and the number of successes X is the random variable of interest. Meanwhile, in the negative binomial distribution, the desired number of successes k is predetermined and the random variable X of interest is the number of Bernoulli trials needed.

To illustrate this difference, observing the number of times a 2 will [redacted]

Pop-Up!

Geometric Distribution

The probability mass function of the discrete random variable X, defined as the number of trials at which the first success occurs, is given by

$$p(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, 3, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

The mean and variance of X are given by $\mu_X = \dfrac{1}{p}$ and $\sigma_X^2 = \dfrac{1-p}{p^2}$, respectively.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

Example 16

Teacher Meldy gave her statistics class a homework to be discussed on the following class meeting. If only 45% of her class diligently do their homework, what is the probability that during the next class meeting,

[redacted]

[redacted]

$$p(x) = \binom{x-1}{2}(.45)^3(.55)^{x-3}, \; x = 3, 4, 5, \ldots .$$

[redacted]

$$P(X = 7) = (.45)(.55)^6 \approx .0125.$$

[redacted]

The Poisson Distribution

A process that examines the number of times an event will occur over a specified time interval or region of space is called a *Poisson experiment*. The number of occurrences of such an event within a specified time interval or region of space is independent of its occurrences in another time interval or region of space. Furthermore, it is nearly impossible for at least two such events to occur simultaneously.

A Poisson experiment has the following characteristics:

1. The occurrences (called *Poisson events*) within a specified time [redacted]

[redacted]

Pop-Up!

Poisson Distribution

A random variable X defined as the number of occurrences of a Poisson event within a specified time interval or size of a region has the *Poisson distribution*, denoted by $X \sim \text{Poi}(\lambda)$, and the following probability mass function:

$$p(x) = \begin{cases} \dfrac{e^{-\lambda} \cdot \lambda^x}{x!}, & x = 0, 1, 2, \ldots, \text{ where } e \approx 2.71828 \\ 0, & \text{otherwise,} \end{cases}$$

where λ is the mean rate of occurrence of the Poisson event within the specified time interval or size of the region.

The Poisson distributed random variable X has mean $\mu_X = \lambda$ and variance $\sigma_X^2 = \lambda$.

The Poisson distribution is also referred to as the *law of small numbers* inasmuch as Poisson events occur rarely. Thus, the Poisson distribution is commonly used to model rare phenomena.

[redacted]

[redacted]

[redacted]

$$[redacted]$$

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[redacted]

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$$[redacted]$$

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