Sampling Distribution of the Sample Mean $\bar{x}$

Sampling is practically done once in order to get a sample that will serve as basis for statistical inferences concerning the population. However, the process may be repeated under basically the same conditions that lead to well-defined possible outcomes. This is why sampling is considered a random experiment.

Appropriate statistics summarizes the data obtained from a sample drawn ramdomly from the population. Examples of such statistics are the sample mean $\bar{x}$, the sample variance s^2, and the sample proportion $\hat{p}$. Because these statistics vary from sample to sample, they are considered random variables. As such, these statistics have probability distributions called *sampling distributions*.

Pop-Up!

A *sampling distribution* is a probability distribution of a statistic.

[redacted]

[redacted]

[redacted]

Possible Samples	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]

Table 8.1 shows that $\overline{x}$ varies from sample to sample indicating that the statistic $\overline{x}$ is indeed a random variable. In chapter 6, a random variable was shown to be characterized by its probability distribution. Constructing the probability distribution of $\overline{x}$ will result to the sampling distribution of $\overline{x}$ shown in table 8.2.

Table 8.2 Sampling Distribution of $\overline{x}$ for Data in Table 8.1

$\overline{x}$	3	4	5	6	7
$p(\overline{x})$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Means, variances, and standard deviations also describe sampling distributions of statistics. You obtain them in a similar manner as that of the means, variances, and standard deviations of any random variable X which you learned in the previous chapter.

[redacted]

[redacted]

[redacted]

The standard deviation of $\bar{x}$ is

$$\sigma_{\bar{x}} = \sqrt{\text{Var}(\bar{x})} = \sqrt{1.6667} \approx 1.2910.$$

This is also referred to as the *standard error* of $\bar{x}$.

Pop-Up!

A *standard error* is the standard deviation of a statistic.

The following can be observed from the mean and variance of both the sampling distribution of $\bar{x}$ and the distribution of X:

- The mean of the sampling distribution of $\bar{x}$ is equal to the mean of the distribution of X, that is $\mu_{\bar{x}} = \mu_X$.
- [redacted]

[redacted]

[redacted]

Pop-Up!

Sampling Distribution of $\bar{x}$: Sampling from a Normal Population

If $\bar{x}$ is the mean of a random sample of size n taken from a normal population with mean μ_X and variance σ_X^2, then the sampling distribution of $\bar{x}$ is normal with mean and variance given by

$$E(\bar{x}) = \mu_X \quad \text{and} \quad \mathrm{Var}(\bar{x}) = \frac{\sigma_X^2}{n},$$

[redacted]

Figure 8.1 on the next page presents graphically results from this important theorem. It can be observed that:

- The shape of the sampling distribution of $\bar{x}$ is the same as the shape of the distribution of X. Both are normally distributed.
- The mean of the sampling distribution of $\bar{x}$ is equal to the mean of [redacted]

[redacted]

[redacted]

[redacted]

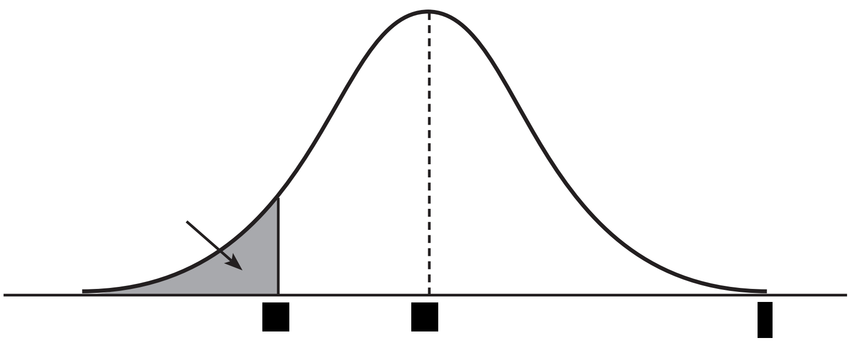
[redacted]

[redacted]

[redacted]

[redacted]

[redacted]



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