Hypothesis Testing About One Population Variance σ^2

Suppose that a random sample of size n is drawn from a normal population. You want to test the null hypothesis that the population variance σ^2 is equal to a specified value σ_0^2, that is

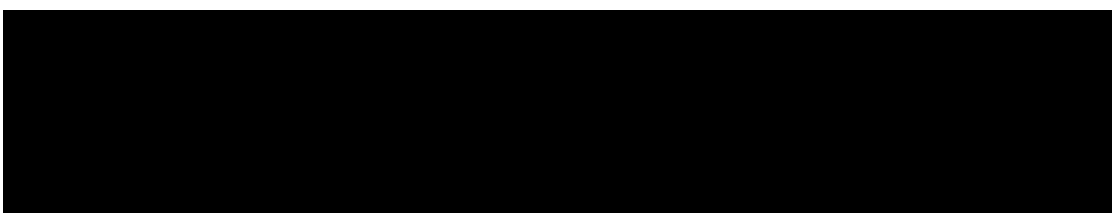
$$H_0: \sigma^2 = \sigma_0^2$$

against one of three possible alternative hypotheses:

$H_a: \sigma^2 \neq \sigma_0^2$ (two-tailed test),

$H_a: \sigma^2 > \sigma_0^2$ (right- or upper-tailed test), aor

$H_a: \sigma^2 < \sigma_0^2$ (left- or lower-tailed test).



Example 5.6

Using a calibration instrument, a researcher needs to confirm whether the standard deviation of the measurements made using his instrument is at most 0.5 units. If not, the instrument needs adjustment. During an experiment, he recorded the following measurements using this instrument on the same object:

6.5, 8.5, 7, 8, 7.5, 7.8, 7, 8.1, 7.2

Does the researcher need to adjust the calibration instrument? Use 5% level of significance.

Solution.

Step 1: H_a: The calibration instrument needs no adjustment, that is $\sigma \leq 0.5$ or $\sigma^2 \leq 0.25$.

H_a: The calibration instrument needs to be adjusted,

$$\sigma > 0.5 \quad \text{or} \quad \sigma^2 > 0.25.$$

Step 2: $\alpha = 0.05$, $\bar{x} = 7.51$, $s = 0.64$, $s^2 = 0.41$, $n = 9$, $\alpha_0^2 = 0.25$.

Step 3: The appropriate chi-square statistic can be computed as follows.

$$\begin{aligned} \chi^2 &= \frac{(n-1)s^2}{\alpha_0^2} \\ &= \frac{(9-1)(0.41)}{0.25} \\ &= 13.12 \end{aligned}$$