

Interval Estimation for the Population Proportion

When the sample size n is sufficiently large ($n\hat{p} > 5$ and $n\hat{q} > 5$), confidence intervals for the population proportion p can be constructed in a similar procedure as that of a confidence interval for population mean μ .

You learned from chapter 3 that the sample proportion \hat{p} can be transformed to a z -score by

$$z = \frac{p - \hat{p}}{\sqrt{\frac{pq}{n}}}$$

Then,

$$P(-Z_{\alpha/2} < z < Z_{\alpha/2}) = 1 - \alpha,$$

where $Z_{\alpha/2}$ is the z -value with an area of $\frac{\alpha}{2}$ in the right-tail of the standard normal curve. By substitution,

$$P\left(-Z_{\alpha/2} < \frac{p - \hat{p}}{\sqrt{\frac{pq}{n}}} < Z_{\alpha/2}\right) = 1 - \alpha.$$

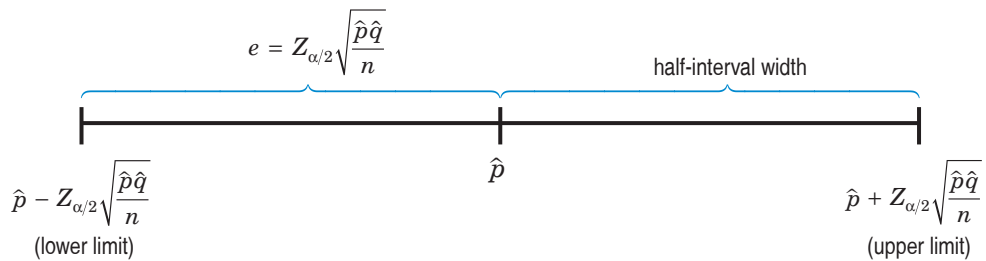
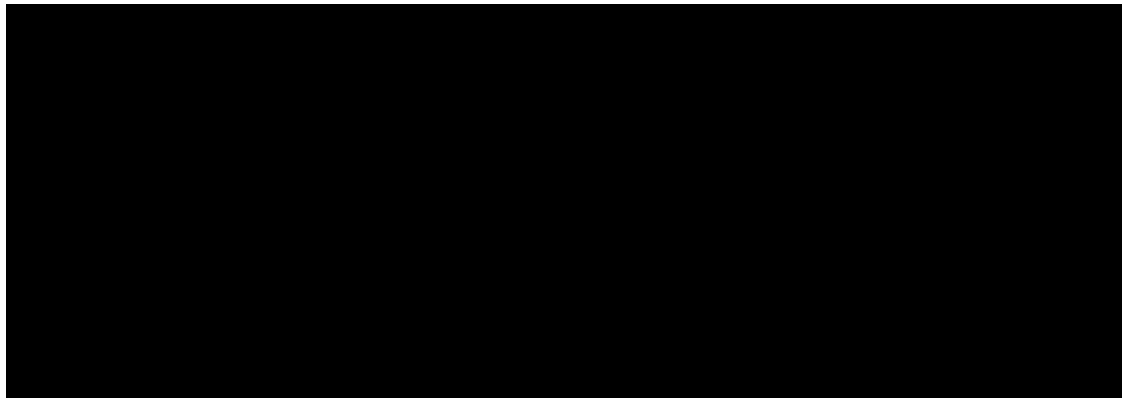


Figure 4.8 A $(1 - \alpha)100\%$ confidence interval for the population proportion p

If an estimate \hat{p} is equal to the population proportion p , that is, both of them are located in the middle of the interval, then \hat{p} estimates p without error. However, most of the time, this is not the case; that is, the error of estimation $|\hat{p} - p| \neq 0$. Using interval estimation, you can be $(1 - \alpha)100\%$ confident that the error will not exceed the maximum error (or margin of error) e , where $e = Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$.

Example 4.7

Recall the situation presented in example 4.6. A survey is conducted to determine the opinions of people on global warming. In a random sample of 150 people, 108 think that global warming is a serious world problem. Construct a 95% confidence interval for the proportion of adults who think that global warming is a very serious world problem.

Solution.

From the solution for example 4.6, the point estimate \hat{p} is 0.72. Hence,

$$\hat{q} = 1 - \hat{p} = 1 - 0.72 = 0.28.$$

Thus,

$$\begin{aligned}\hat{p} - z_{0.05/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} &< p < \hat{p} + z_{0.05/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} \\ 0.72 - z_{0.025} \sqrt{\frac{(0.72)(0.28)}{150}} &< p < 0.72 + z_{0.025} \sqrt{\frac{(0.72)(0.28)}{150}} \\ 0.72 - 1.96(0.0367) &< p < 0.72 + 1.96(0.0367) \\ 0.72 - 0.0719 &< p < 0.72 + 0.0719 \\ 0.6481 &< p < 0.7919.\end{aligned}$$

You are 95% confident that the error will not exceed 0.0719 (or 7.19%), and the true proportion of adults who think that global warming is a very serious world problem is between 0.6481 (or 64.81%) and 0.7919 (or 79.19%).

